Exam. Code : 211002
Subject Code : 5544

M.Sc. (Mathematics) 2nd Semester

DIFFERENTIAL AND INTEGRAL EQUATIONS

Paper—MATH-565

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Candidate to attempt **TWO** questions from each unit. Each question carries equal marks.

UNIT—I

1. Prove that the general solution of linear differential equation $Pp + Qq = R$ is of the form $F(u, v) = 0$, where $F(u, v)$ is an arbitrary function of $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ which form a solution of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

2. Find the equation of the integral surface of the differential equation $2y(z - 3)p + (2x - z)q = y(2x - 3)$ which passes through the circle $z = 0$, $x^2 + y^2 = 2x$.

3. Find the surface which is orthogonal to the one parameter system $z = cxy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 = a^2$, $z = 0$.

4. Use Charpit's method to solve the partial differential equation $(p^2 + q^2)y = qz$.

UNIT—II

5. If f and g are arbitrary functions of their respective arguments, show that $u = f(x - vt + i\alpha y) + g(x - vt - i\alpha y)$ is a solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$, provided

$$\alpha = \sqrt{1 - \frac{v^2}{c^2}}.$$

6. Solve the equation :

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}.$$

7. Reduce the following partial differential equation into canonical form and hence solve it :

$$y^2 \frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}.$$

8. Solve the wave equation r = t by Monge's method.

UNIT—III

9. Solve the Laplace equation in spherical coordinates by method of separation of variables.

10. The ends A and B of a rod, 10 cm in length are kept at temperature 0ºC and 100ºC, respectively until the steady state condition prevails. Suddenly the temperature at the end A is increased to 20ºC and at the end B is decreased to 60ºC. Find the temperature distribution in rod at time t.

11. Obtain the appropriate solution of the radio equation $\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 v}{\partial t^2}$ appropriate to the case when a periodic e.m.f. $V_0 \cos pt$ is applied at the end $x = 0$ of the line.

12. Solve the following heat conduction equation using Fourier transforms :

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty, t > 0,$$

subject to the initial and boundary conditions as :

$u(x, 0) = f(x), -\infty < x < \infty$ and $u(x, t) \to 0$ and $\partial u/\partial x \to 0$ as $x \to \infty$.

UNIT—IV

13. Explain the relation between linear non homogeneous differential equation and Volterra integral equation.

14. Explain the method of successive substitution for the solution of Volterra integral equation.

15. Define reciprocal function. If K(x, t) is real and continuous in R, there exists a reciprocal function k(x, t), provided that $M(b - a) < 1$, where M is maximum of K(x, t) in R.

16. Solve the integral equation $u(x) = x + \int_0^x (t - x)\, u(t)dt$.

UNIT—V

17. Solve the Fredholm equation :

$$u(x) = e^x - \frac{e-1}{2} + \frac{1}{2}\int_0^1 u(t)dt.$$

18. Explain the method of successive approximations for the solution of Fredholm integral equation.

19. If K(x, t) is non-zero real and continuous in R and f(x) is non-zero real and continuous I. A function k(x, t) is reciprocal to K(x, t) exists then the Fredholm integral equation $u(x) = f(x) + \int_a^b k(x, t)\, u(t)dt$ has the solution of the form $u(x) = f(x) - \int_a^b K(x, t)\, f(t)dt$.

20. Compute $D(\lambda)$ for the integral equation :

$$u(x) = f(x) + \lambda \int_0^{\pi} \sin x\, u(t)dt.$$